

MATH MAGIC: A 6th Grade Unit

TEACHER'S GUIDE

Alignment to Washington 2008 Mathematics Standards for Grade 6

- 6.1 Core Content:** Multiplication and division of fractions and decimals
6.1.H
- 6.2. Core Content:** Mathematical expressions and equations
(Operations, Algebra)
6.2.A, 6.2.C
- 6.6. Core Processes:** Reasoning, problem solving, and
communication
6.6.A, 6.6.B, 6.6.C, 6.6.D, 6.6.E, 6.6.F, 6.6.G, 6.6.H

Vocabulary

Terms first used at Grade 6 in the Measurement of Student Progress (MSP) or in the Association of Educational Service Districts (AESD) 6th Grade word list are identified in bold print.

Algebraic expression
Finite
Relationship

Conclude
Negative **integers**
Solution

Conjecture
Operations
Variable

WARM UP 1

Part A

Think of a number between 1 and 100. What is your number? ____
Double the number.
Add 6.
Divide your answer by two.
Subtract your original number.
What is your answer? ____

Part B

Pick two new numbers and then fill in the chart.

	Trial 2	Trial 3
Pick a number between 1 and 100.		
Double the number.		
Add 6.		
Divide your answer by two.		
Subtract the original number that you picked.		

Part C

Compare your chart with your partner's chart. What do you notice?

Teacher's Guide: Grade 6

Each unit targets the WA 2008 Mathematics Core Content and Core Process Standards.

This resource includes all of the 6th grade AESD and MSP terms. Instructional suggestions for vocabulary acquisition are included in Section I, Chapter C, Part 1, of this book.

Each unit includes one or more Warm Up Problems, a Problem Solving Task and one or more Extension Problems.

Visual solutions are encouraged, throughout.

Warm Up 1 Suggestions



TO HELP STUDENTS UNDERSTAND THE PROBLEM:

- Select a student to experience this activity in front of the class. Then select another student to do the same, using a different number.
- Ask the class if they have a prediction about what the “answer” will always be. (6.6.H)
- Then ask students to test their predictions with different numbers. (6.6.H)

A Solution

Although different numbers are picked, the answer is always “3”.

WARM UP 2

Now make a pictorial representation of the problem from Warm Up 1. Use a single tile to represent your “mystery number”. Use another color of tiles to represent numbers that you add to or subtract from your mystery number.

Think of a number between 1 and 100.

Double the number.

Add 6.

Divide your answer by two.

Subtract the original number.

What is your answer? _____

Warm Up 2 Suggestions



TO HELP STUDENTS UNDERSTAND THE PROBLEM:

Ask:

- What do you know about the problem and the questions? (6.6.A)
- What does “pictorial representation” mean? (6.6.B)
- What does, “Use a single tile to represent your mystery number” mean? How many tiles will I use if my mystery number is “3”? (6.6.B)
- How is this problem similar to Warm Up 1? How is it different? (6.6.C)
- Should you use a different color of tile or different type of counter to represent the “mystery number” and the numbers you add and subtract? Why? Why not? (6.6.C) *It is less confusing to use a different color or type of representation for the “unknown mystery number”.*

Note: In Warm Up 3, students connect pictorial representations to algebraic expressions.

Teacher’s Guide: Grade 6 (continued)

The coding (6.6.H, etc.) shows how the instructional suggestions are designed to match the WA Core Process Standards for Grade 6.

Possible solutions are included when one answer is appropriate.

Display Files with Virtual Manipulatives are available to accompany this unit.

Each unit includes black line masters of each problem with permission for one classroom teacher to make copies for his/her students.

PROBLEM SOLVING TASK

Part A

Follow the directions in the first column for three trials with different “mystery numbers” of your choice. Record what is happening to each number as it goes through the steps.

Part B

Draw a pictorial representation of each part of the directions.

Part C

Using n for the “mystery number”, write algebraic expressions that represent each line of the directions.

Directions	Trial 1	Trial 2	Trial 3	Pictorial Representation	Algebraic Expression
Pick any “mystery number” between 1 and 100.					
Add 4 to it.					
Add the “mystery number” to your sum.					
Divide by two.					
Subtract your “mystery number”.					
Answer:					

Problem Solving Task Suggestions



TO HELP STUDENTS UNDERSTAND THE PROBLEM:

Ask:

- What do you know about the problem? (6.6.A)
- What information is relevant? (6.6.B)
- Is any information missing? (6.6.B)
- In what ways is this problem similar to the Warm Ups? How is it different? (6.6.C)

Then tell students to select a method or a way to begin. (6.6.C)

Teacher’s Guide: Grade 6 (continued)

Pages have been reduced in size and combined for this sample folder.



AS STUDENTS WORK:

Circulate to observe, listen to student discourse and ask questions as needed. Select several different solutions for sharing when students are finished.

Common misconceptions or errors:

- Inappropriate representation, especially with the “divide by 2” direction.
- Forgetting to work with only one of the halves after dividing by two.
- Inappropriate algebraic expressions, especially with the “divide by 2” direction.
- Not matching the pictorial representation to the algebraic expressions. Students may skip a step in one of the columns and get out of sequence.



WHEN STUDENTS ARE FINISHED:

Analyze and evaluate whether the solution is reasonable and correct. Have students carefully read the problem again. Then, in pairs, review:

- Did you use all of the directions? (6.6.D)
- Explain to your partner how you solved each part. (6.6.D, 6.6.E)
- Check to see if you can find any errors. (6.6.D)

Invite the groups you selected to share their work.

- Ask students to explain how and why they solved the problem the way they did. (6.6.E, 6.6.F)
- Through questioning, help students discover any errors themselves.
- Ask the class if they agree or disagree and why. (6.6.G)
- When a conceptually rich solution or explanation is presented, ask another student to summarize the solution and explain the reasoning. (6.6.G)
- Ask questions to focus discussion on understanding the relationship between numerical information, pictorial representations and algebraic expressions.

Ask students to make a conjecture about what the answer will always be, no matter what number is chosen as the “mystery number.” (6.6.H) Ask:

- How could you test your conjecture? (*Allow time for testing and discussion.*)
- If you tested 500 different cases, could you conclude that you have proven the conjecture to be correct? (*No amount of “testing” will ever prove a conjecture correct, unless there are only a finite number of cases and you test them all.*)

(Suggestions continue...)



Teacher’s Guide: Grade 6 (continued)





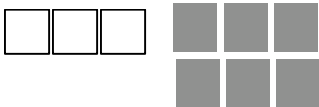


Although the problems in this unit are scaffolded, most units provide problems in MSP format. This program often encourages multiple ways to find solutions. For most problems, we encourage sharing the most concrete solutions first, so that all students can understand. We ask that the most abstract or the most sophisticated solutions be shared last. With that kind of scaffolding, all students can be successful.

EXTENSION 1

Part A

Complete the following table.

Let  = a letter variable of your choice
 = 1

Pictorial Representation	Written Direction	Algebraic Expression
	Pick a variable to represent 	
	Add 1 to your number	
	Multiply by 3	
	Add 3	
	Divide by ____	
	Subtract ____	
	Result:	

Part B

Test your algebraic expressions with several “mystery numbers” of your choice. Use your mystery number to evaluate each expression.

Extension 1 Suggestions



HELP STUDENTS UNDERSTAND THE PROBLEM:

Ask questions about the problem and about solution strategies. (6.6.A, 6.6.B, 6.6.C, 6.6.H)



WHEN STUDENTS ARE FINISHED:

Analyze and evaluate whether the solution is reasonable and correct. If there is any variance in solutions, invite a group you selected to share their work. (6.6.D, 6.6.E, 6.6.F, 6.6.G, 6.6.H)

Note: Extension 2 allows students to try the same procedure with a fraction or a decimal.

Teacher's Guide: Grade 6 (continued)

Extension problems are included in each unit for students who are ready for additional challenges.

The range of difficulty, from the first Warm Up to the hardest Extension, provides appropriate challenges for students of diverse skill levels.

G.4 Math Magic Rubric*	
Level	
4 Beyond Proficient	In addition to the requirements for a 3, the student shows “in-depth inferences and applications that go beyond what was taught.” Advanced understanding may be shown in connections, generalizations not required by the task, higher order approaches to the task, insightful methodology, or other ways.
3 Proficient	The response has “no major errors or omissions” and shows that the student has: <ul style="list-style-type: none"> • Written a mathematical expression... with (a) variable to represent information in a table; • Demonstrated understanding of each piece of the task (numeric, tabular, and algebraic) as well as the underlying mathematical concepts; and, • Completed the chart that shows a result of 2 each time.
2 Partially Effective	The response shows “no major errors or omissions regarding the simpler details and processes” and indicates that the student has: <ul style="list-style-type: none"> • Used the variable n (or another letter) for the mystery number; and, • Translated the directions into at least one other correct algebraic expression. <p>However, the work shows “major errors or omissions regarding the more complex ideas and processes.” The response may include only a partially completed chart, or the chart may be only partially accurate.</p>
1 Limited	The response shows that the student demonstrates a limited understanding of some of the simpler details and processes of the task. <p><i>Note: In your classroom, score a 1 if a student, with help, shows “partial understanding of some of the simpler details and some of the more complex ideas and processes.”</i></p>
0 Insufficient	The response shows that the student, even with help, shows no understanding or demonstrates no skill related to this topic.

**Teacher’s Guide:
Grade 6
(continued)**

All quotes and many ideas for the rubrics are from WA ESD/Marzano rubrics.

Line(s) in bold italics are taken directly from the WA 2008 Mathematics Standards.

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Line(s) in bold are taken directly from the WA 2008 Mathematics Standards.

G.9 MATH MAGIC
Sample and Scored Commentary

Sample (S) #2: Score 2 (Partially Effective)

The response shows no major errors or omissions regarding the simpler details and processes. The variable n is used for the mystery number, and the student correctly writes the expression " $n + 4$ " in the second step and " $2n + 4$ " in the third step. The pictorial representations are correct throughout, as is the answer of "2". However, the algebraic expressions in the fourth and fifth steps, while inventive, are incorrect. Instead of using " n " for the mystery number, the work shows " $w \div 2$ " and " $w - n$ " and defines " w " as "a working answer, an answer you get while trying to find the real answer."

Sample (S) # 2
Score: 2

PKD

Part A







Follow the directions in the first column for three trials with different "mystery numbers" of your choice. Record what is happening to each number as it goes through the steps.

Part B

Draw a pictorial representation of each part of the directions.

Part C

Using n for the "mystery number", write algebraic expressions that represent each line of the directions.

Directions	Trial 1	Trial 2	Trial 3	Pictorial Representation	Algebraic Expression
Choose a mystery number	4	2	6		n
Add 4 to it	8	6	10		$n + 4 =$
Add the mystery number to your sum	12	8	16		$2n + 4$
Divide by two	6	4	8		$w \div 2$
Subtract your mystery number	2	2	2		$w - n$
Answer	2	2	2		2

*△ = mystery #
○ =
n = Mystery #
w = a working answer, an answer you get while trying to find the real answer*

**Teacher's Guide:
Grade 6
(continued)**

Each Problem Solving Task has several samples of student work with scored commentaries based on an individual 4-point rubric.

The samples, commentaries and individual rubrics assist teachers in evaluating the work of their students.

The scored samples and commentaries also help students learn how to evaluate their own work on Problem Solving Tasks.

“FIX IT!”
G.9 MATH MAGIC

DIRECTIONS: CHANGE OR ADD TO THE WORK BELOW TO IMPROVE THE FINAL SCORE.
T/PJS

Part A

Follow the directions in the first column for three trials with different “mystery numbers” of your choice. Record what is happening to each number as it goes through the steps.

Part B

Draw a pictorial representation of each part of the directions.

Part C

Using n for the “mystery number”, write algebraic expressions that represent each line of the directions.

Directions	Trial 1	Trial 2	Trial 3	Pjs	
				Pictorial Representation	Algebraic Expression
Pick any “mystery number” between 1 and 100.	99	37	1		
Add 4 to it.	103	41	5		$N+4$
Add the “mystery number” to your sum.	202	78	6		$NN+4$
Divide by two.	101	39	3		$NN+4 \div 2$
Subtract your “mystery number”.	99	37	1		$NN+4 \div 2 - N$
Result:	2	2	2	2	2

TEACHER SELF-EVALUATION FORM

Check the strategies you used in this unit and note what you said or did.

WAYS TO HELP STUDENTS MAKE SENSE OF PROBLEM SOLVING	✓	WHAT I SAID OR DID
• Be aware of the mathematics embedded in each unit—Warm Up(s), Task and Extension(s)—so you can engage students in discussions that will deepen mathematical understanding.		
• Encourage visualization of solutions, especially with the use of manipulatives.		
• Encourage students to rely on their own and each other’s thinking.		

(Form continues...)

**Teacher’s Guide:
Grade 6
(continued)**

After a class has completed a unit, the “Fix It!” activity can give students practice in revising and improving a work sample.

The job of the student in these exercises is to analyze what makes sense in the sample and what needs changing, and then to fix and complete the solution.

The Teacher Self-Evaluation Form is included at the end of each unit. It can assist teachers as they learn to incorporate these instructional strategies into their daily problem-solving lessons.